

Examen de Matemáticas – 4º de ESO – Opción B

1. Resolver las siguientes operaciones con fracciones, simplificando en todo momento los pasos intermedios y el resultado. **(2 puntos; 1 punto por apartado)**

$$\text{a) } \frac{\left(\frac{2}{5} - \frac{1}{3}\right) : \left(1 - \frac{1}{3} \cdot \frac{6}{5}\right)}{1 - \left(\frac{2}{3} \cdot \frac{1}{3} - 1\right)} + 1 = \quad \text{b) } \frac{3 + \frac{3}{2 + 1/2}}{\frac{1}{15} + \left(\frac{3}{2} - \frac{1}{3} \cdot \frac{5}{2}\right) \cdot \frac{1}{5}} =$$

2. Realiza las siguientes operaciones con potencias y simplifica el resultado todo lo posible (*se puede dejar el resultado en forma de potencia*). **(2 puntos; 1 punto por apartado)**

$$\text{a) } \frac{4^2 \cdot 2^{-2} \cdot 9^{-3} \cdot 6^3}{12 \cdot 3^{-3} \cdot 2 \cdot 3^{-3}} = \quad \text{b) } \frac{\left(\frac{5}{4}\right)^{-3} \cdot \left(\frac{25}{4}\right)^3}{5^3 \cdot \left(\frac{2}{5}\right)^{-2} \cdot \left(\frac{5}{2}\right)^{-3} \cdot \left(\frac{4}{5}\right)^2} =$$

3. Opera y simplifica extrayendo factores siempre que sea posible (recuerda que has de factorizar los números que no sean primos): **(1 punto; 0,5 puntos por apartado)**

$$\text{a) } \sqrt{16} \sqrt[5]{64} = \quad \text{b) } 3\sqrt{2} + 4\sqrt{8} - \sqrt{32} + \sqrt{50} =$$

4. Racionaliza: **(1 punto; 0,5 puntos por apartado)**

$$\text{a) } \frac{6}{\sqrt[3]{3}} = \quad \text{b) } \frac{9}{\sqrt{5} - \sqrt{2}} =$$

5. Resuelve las siguientes ecuaciones: **(4 puntos; 1 punto por apartado)**

$$\text{a) } \frac{1}{3}(x+2) - \frac{1}{5}(2x-3) = 4 - \frac{2x}{15} \quad \text{b) } \frac{x+5}{3} + \frac{x-3}{2} = \frac{x+5}{5} - \frac{3x}{15}$$

$$\text{c) } \frac{x(x+1)}{5} = 2x^2 - 4x \quad \text{d) } \left(\frac{3}{2}x - 2\right)^2 - (x-1)(x+1) = -2$$

$$\begin{aligned} \textcircled{1} \text{ a) } & \frac{\left(\frac{2}{5} - \frac{1}{3}\right) : \left(1 - \frac{1}{3} \cdot \frac{6}{5}\right)}{1 - \left(\frac{2}{3} \cdot \frac{1}{3} - 1\right)} + 1 = \frac{\left(\frac{6}{15} - \frac{5}{15}\right) : \left(1 - \frac{6}{15}\right)}{1 - \left(\frac{2}{9} - 1\right)} + 1 = \\ & = \frac{\frac{1}{15} : \left(1 - \frac{2}{5}\right)}{1 - \left(\frac{2}{9} - \frac{9}{9}\right)} + 1 = \frac{\frac{1}{15} : \left(\frac{5}{5} - \frac{2}{5}\right)}{1 - \left(-\frac{7}{9}\right)} + 1 = \frac{\frac{1}{15} : \frac{3}{5}}{1 + \frac{7}{9}} + 1 = \\ & = \frac{\frac{5}{45}}{\frac{9}{9} + \frac{7}{9}} + 1 = \frac{\frac{1}{9}}{\frac{16}{9}} + 1 = \frac{1 \cdot 9}{9 \cdot 16} + 1 = \frac{1}{16} + 1 = \frac{1}{16} + \frac{16}{16} = \underline{\underline{\frac{17}{16}}} \end{aligned}$$

$$\begin{aligned} \text{b) } & \frac{3 + \frac{3}{2 + 1/2}}{\frac{1}{15} + \left(\frac{3}{2} - \frac{1}{3} \cdot \frac{5}{2}\right) \cdot \frac{1}{5}} = \frac{3 + \frac{3}{5/2}}{\frac{1}{15} + \left(\frac{3}{2} - \frac{5}{6}\right) \frac{1}{5}} = \frac{3 + \frac{6}{5}}{\frac{1}{15} + \left(\frac{9}{6} - \frac{5}{6}\right) \frac{1}{5}} = \\ & = \frac{\frac{15}{5} + \frac{6}{5}}{\frac{1}{15} + \frac{4}{6} \cdot \frac{1}{5}} = \frac{\frac{21}{5}}{\frac{1}{15} + \frac{4}{30}} = \frac{\frac{21}{5}}{\frac{2}{30} + \frac{4}{30}} = \frac{\frac{21}{5}}{\frac{6}{30}} = \frac{\frac{21}{5}}{\frac{1}{5}} = \underline{\underline{21}} \end{aligned}$$

$$\textcircled{2} \text{ a) } \frac{4^2 \cdot 2^{-2} \cdot 9^{-3} \cdot 6^3}{12 \cdot 3^{-3} \cdot 2 \cdot 3^{-3}} = \frac{2^4 \cdot 2^{-2} \cdot 3^{-6} \cdot 2^3 \cdot 3^3}{2^2 \cdot 3 \cdot 3^{-3} \cdot 2 \cdot 3^{-3}} = \frac{2^5 \cdot 3^{-3}}{2^3 \cdot 3^{-5}} = 2^2 \cdot 3^2 = \underline{\underline{36}}$$

$$\begin{aligned} \text{b) } & \frac{\left(\frac{5}{4}\right)^{-3} \cdot \left(\frac{25}{4}\right)^3}{5^3 \cdot \left(\frac{2}{5}\right)^{-2} \left(\frac{5}{2}\right)^{-3} \left(\frac{4}{5}\right)^2} = \frac{\frac{5^{-3}}{2^{-6}} \cdot \frac{5^6}{2^6}}{5^3 \cdot \frac{2^{-2}}{5^{-2}} \cdot \frac{5^{-3}}{2^{-3}} \cdot \frac{2^4}{5^2}} = \frac{\frac{5^3}{2^0}}{\frac{5^0 \cdot 2^2}{5^0 \cdot 2^{-3}}} = \\ & = \frac{5^3 \cdot 2^{-3}}{2^2} = 5^3 \cdot 2^{-5} = \frac{5^3}{2^5} = \underline{\underline{\frac{125}{32}}} \end{aligned}$$

$$\begin{aligned} \textcircled{3} \text{ a) } & \sqrt{16} \sqrt[5]{64} = \sqrt{2^4} \cdot \sqrt[5]{2^6} = \sqrt{5 \sqrt{(2^4)^5 \cdot 2^6}} = \\ & = \sqrt{5 \sqrt{2^{20} \cdot 2^6}} = \sqrt[10]{2^{26}} = 2^2 \cdot \sqrt[10]{2^6} = 4 \cdot \sqrt[5]{2^3} = \underline{\underline{4 \sqrt[5]{8}}} \end{aligned}$$

$$\begin{aligned} \text{b) } & 3\sqrt{2} + 4\sqrt{8} - \sqrt{32} + \sqrt{50} = 3\sqrt{2} + 4\sqrt{2^3} - \sqrt{2^5} + \sqrt{2 \cdot 5^2} = \\ & = 3\sqrt{2} + 4 \cdot 2\sqrt{2} - 2^2\sqrt{2} + 5\sqrt{2} = 3\sqrt{2} + 8\sqrt{2} - 4\sqrt{2} + 5\sqrt{2} = \\ & = (3 + 8 - 4 + 5)\sqrt{2} = \underline{\underline{12\sqrt{2}}} \end{aligned}$$

$$\textcircled{4} \text{ a) } \frac{6}{\sqrt[3]{3}} = \frac{6 \sqrt[3]{3^2}}{\sqrt[3]{3} \sqrt[3]{3^2}} = \frac{6 \sqrt[3]{9}}{\sqrt[3]{3^3}} = \frac{6 \sqrt[3]{9}}{3} = \underline{\underline{2 \sqrt[3]{9}}}$$

$$\text{b) } \frac{9}{\sqrt{5}-\sqrt{2}} = \frac{9(\sqrt{5}+\sqrt{2})}{(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2})} = \frac{9(\sqrt{5}+\sqrt{2})}{\sqrt{5^2}-\sqrt{2^2}} = \frac{9(\sqrt{5}+\sqrt{2})}{3} = \underline{\underline{3(\sqrt{5}+\sqrt{2})}}$$

$$\textcircled{5} \text{ a) } \frac{1}{3}(x+2) - \frac{1}{5}(2x-3) = 4 - \frac{2x}{15} \Rightarrow (\text{multiplicando por } 15)$$

$$5(x+2) - 3(2x-3) = 60 - 2x \Rightarrow 5x+10 - 6x+9 = 60 - 2x$$

$$\Rightarrow -x+19 = 60 - 2x \Rightarrow -x+2x = 60-19 \Rightarrow \underline{\underline{x=41}}$$

$$\text{b) } \frac{x+5}{3} + \frac{x-3}{2} = \frac{x+5}{5} - \frac{3x}{15} \Rightarrow (\text{multiplicando por } 30)$$

$$10(x+5) + 15(x-3) = 6(x+5) - 6x \Rightarrow$$

$$\Rightarrow 10x+50+15x-45 = 6x+30-6x \Rightarrow 25x+5 = 30$$

$$\Rightarrow 25x = 30-5 \Rightarrow 25x = 25 \Rightarrow x = \frac{25}{25} \Rightarrow \underline{\underline{x=1}}$$

$$\text{c) } \frac{x(x+1)}{5} = 2x^2 - 4x \Rightarrow (\text{multiplicando por } 5)$$

$$x(x+1) = 10x^2 - 20x \Rightarrow x^2 + x = 10x^2 - 20x \Rightarrow$$

$$\Rightarrow x^2 + x - 10x^2 + 20x = 0 \Rightarrow -9x^2 + 21x = 0 \Rightarrow$$

$$\Rightarrow x(-9x+21) = 0 \quad \left\{ \begin{array}{l} \underline{\underline{x=0}} \\ \end{array} \right.$$

$$-9x+21=0 \Rightarrow -9x=-21$$

$$\Rightarrow x = \frac{-21}{-9} \Rightarrow \underline{\underline{x = \frac{7}{3}}}$$

$$\text{d) } \left(\frac{3}{2}x-2\right)^2 - (x-1)(x+1) = -2 \Rightarrow$$

$$\Rightarrow \frac{9}{4}x^2 - 6x + 4 - (x^2-1) = -2 \Rightarrow (\text{multiplicando por } 4)$$

$$\Rightarrow 9x^2 - 24x + 16 - 4x^2 + 4 = -8 \Rightarrow 5x^2 - 24x + 28 = 0$$

$$\Delta = b^2 - 4ac = (-24)^2 - 4 \cdot 5 \cdot 28 = 576 - 560 = 16$$

$$x = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{24 \pm \sqrt{16}}{10} = \frac{24 \pm 4}{10} = \left\{ \begin{array}{l} \frac{28}{10} = \underline{\underline{\frac{14}{5}}} \\ \frac{20}{10} = \underline{\underline{2}} \end{array} \right.$$